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Models for Vapor-Phase and Liquid-Phase Mass Transfer on Distillation Trays

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Three mass transfer models are developed for the vapor phase. The first model assumes a rigid interface between the liquid and vapor and yields the Schmidt number as the correlating variable. The other two models represent a free interface with the penetration theory as the mechanism for one model and eddy diffusion as the mechanism for the other model. Both free interface models correlate experimental data within the error of the data. A liquid-phase mass transfer model is developed for a free interface and the penetration theory as the mass transfer mechanism. Design equations are developed from this analysis.

The AIChE Bubble-Tray Design Manual (*1*) proposes models for mass transfer in the vapor and liquid phases of bubble-cap trays. The models are based upon experimental data obtained as part of this research program. Correlation equations for the vapor and liquid phases are

$$N_G = (0.776 + 0.116\,W - 0.29\,F + 0.0217\,L)/(N_{Sc})^{0.5} \qquad (1)$$

$$N_L = 6180\ (0.26\,F + 0.15)\ t_L(D_L)^{0.5} \qquad (2)$$

Application of the AIChE design method has indicated that liquid-phase-controlled binary systems operating at low liquid rates have much lower efficiencies than are predicted by the design method (*2*). Hughmark (*3*) analyzed the AIChE data and proposed a single surface renewal model for the vapor phase with the penetration theory. A more recent analysis (*4*) of University of Michigan humidification data (*5*) with experimental interfacial areas from the work of Porter, King, and Varshney (*6*) indicates that many surface renewals occur during the gas residence time on the tray. The objective of this paper is to consider additional models for the vapor and liquid phases and to evaluate these models with the experimental data from the AIChE program.

GAS OR VAPOR PHASE

The experimental interfacial area data of Porter, King, and Varshney indicate that the area per unit volume of total holdup is essentially constant for the F range of 0.3 to 0.8 which is the range of the data. It can be postulated that the gas or vapor passes from the cap slots as a turbulent jet and that the mass transfer area corresponds to the interface between the jet and the liquid. The assumption of a continuous turbulent stream for the gas was considered in a prior communication (*4*) and the experimental data were shown to be consistent with this assumption. Byers and King (*7*) have shown that the additive resistance principle for two phases should be closely obeyed for concurrent flow at an interface. The jet model represents this condition, so the two-film resistance model used in the AIChE work should be applicable to this analysis.

Schmidt Number Model

Equation (1) is a Schmidt number model which is an empirical representation of data obtained at the University of Delaware for the ammonia-air-water system at atmospheric pressure and for the acetone-benzene system at 7.4 to 79.7 lb./sq.in.abs. A Schmidt number model can also be derived with the penetration theory.

Beek and Bakker (*8*) considered mass transfer between gas and liquid phases which are in fully developed concurrent flow and are infinite in extent. Their analysis showed that the solution for short exposure times, where the slope in the velocity profile is insignificant with respect to the interfacial velocity, approaches the penetration model. The prior communication (*4*) shows the derivation of a dimensionless gas-phase velocity for the penetration model, with the assumptions

1. Momentum transfer from the gas to the liquid can be represented as that to a rigid interface.
2. Momentum transfer for a distillation tray is represented by $\rho_G u_G^2$.

The dimensionless velocity is

$$u_c^+ = \frac{F}{2}\sqrt{\frac{\pi t_G'}{\mu}} \tag{3}$$

Penetration theory representation of the mass transfer is

$$k_G = \frac{2}{RT}\sqrt{\frac{D_G}{\pi t_G'}} \tag{4}$$

and $\bar{a}$ is defined as the interfacial area per unit of bubbling area on the tray:

$$k_G\bar{a} = \frac{2\bar{a}}{RT}\sqrt{\frac{D_G}{\pi t_G'}} \tag{5}$$

and

$$k_G\bar{a} = \frac{N_G G}{P} \tag{6}$$

so combining Equations (3), (5), and (6), we get

$$N_G = \frac{\bar{a}FP}{u_c^+ G(RT)}\sqrt{\frac{D_G}{\mu}} \tag{7}$$

and substitution of $F = u_G\sqrt{\rho G}$ and $G = u_G\,P/RT$ in Equation (7) yields

$$N_G = \frac{\bar{a}}{u_c^+\sqrt{N_{Sc}}} \tag{8}$$

Equation (8) is observed to be of the same form as Equation (1), with $\bar{a}/u_c^+$ representing the empiricism for W, F, and L.

The University of Michigan humidification data can be used to evaluate this model in addition to the University of Delaware data used to develop Equation (1). Humidification data are reported for air-water, helium-water, Freon 12-water, nitrogen-isobutanol, helium-isobutanol, and helium-MIBK. Identical liquid flows and weir heights were used and data are available for the F range of 0.77 to 0.92. Figure 1 shows N_G as a function of the Schmidt number for data corrected to $F = 0.91$. The Schmidt number correlation as shown by Figure 1 is observed to be a rather poor fit of the data, as the data do not show a square root response for the Schmidt number. Figure 2 shows a plot of $N_G\sqrt{N_{Sc}}$ versus F for the range of F from 0.9 to 2.7 for the AIChE data as a test of Equations (1) and (8). These data represent weir heights of 1½ to 4¼ in. and liquid rates of 7 to 38 g.p.m./ft. of tray width. Figure 2 shows a reasonable correlation for $N_G\sqrt{N_{Sc}}$ as a function of F for the Delaware data which corresponds to the relatively narrow range of 0.7 to 0.9 for the Schmidt

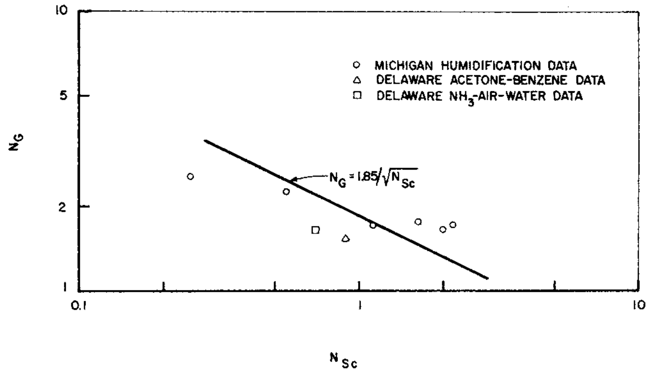

Fig. 1. Schmidt number model.

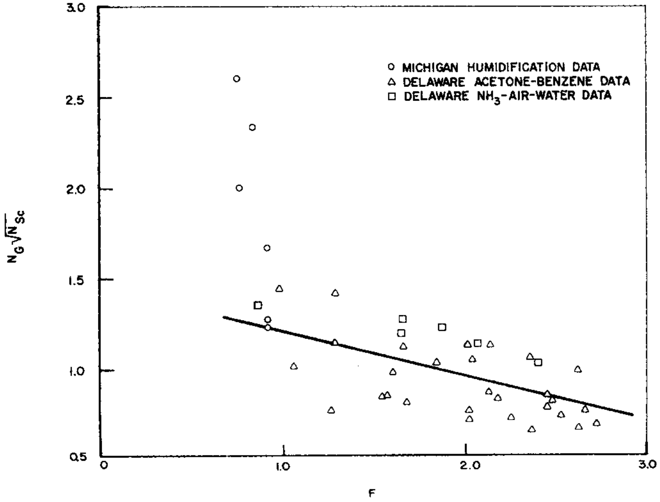

Fig. 2. Schmidt number model.

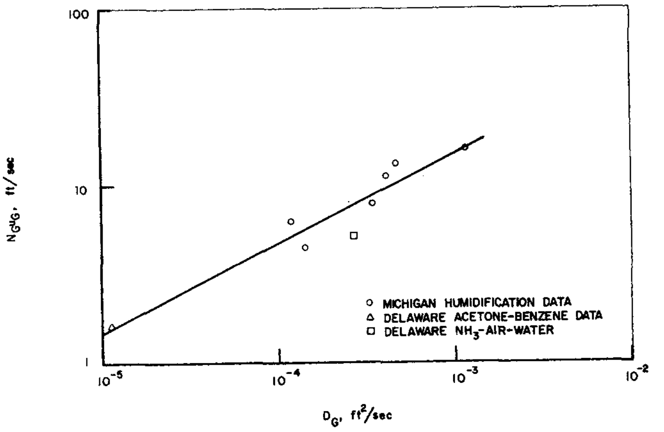

Fig. 3. Molecular diffusion model; $F = 0.91$.

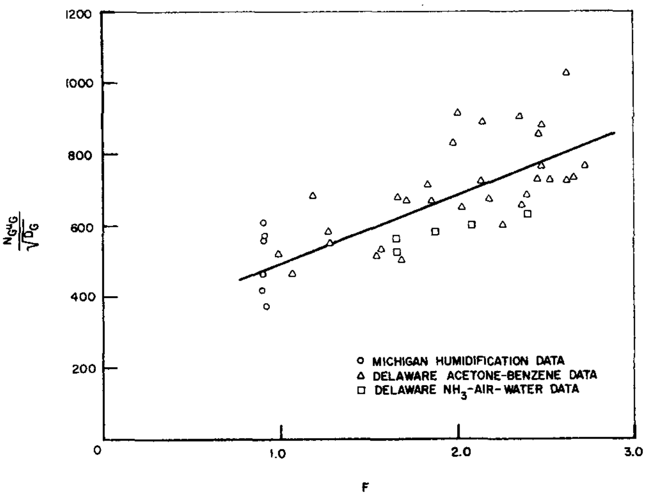

Fig. 4. Molecular diffusion model.

number. There does not appear to be a definite correlation with weir height or liquid flow as indicated by Equation (1). This indicates that the F factor is the controlling variable for N_G and that the liquid effects are not a major contribution. Thus the Schmidt number correlation for the Delaware data does not appear to extrapolate to the wider range of Schmidt numbers represented by the humidification data.

Molecular Diffusion Model

The Schmidt number model is based on the assumption of momentum transfer for a rigid interface between the gas and liquid. A free interface exists for a jet of this type so the second model eliminates the rigid interface assumption.

Combination of Equations (5) and (6) with substitution for G gives

$$N_G = \frac{2\bar{a}}{u_G}\sqrt{\frac{D_G}{\pi t_G'}}$$

The group $2\bar{a}/\sqrt{\pi t_G'}$ may be considered as a function of F so the equation can be written in the form

$$N_G = \frac{2\bar{a}}{\sqrt{\pi t_G'}}\sqrt{\frac{D_G}{u_G}} \tag{9}$$

This is designated as the molecular diffusion model because N_G is shown as a function of the molecular diffusivity rather than the Schmidt number as shown by the first model.

Figure 3 shows $N_G u_G$ as a function of molecular diffusivity for the data with $F = 0.91$ in accordance with Equation (9). The data are observed to fit this model better than the Schmidt number model shown by Figure 1 and show a square root response for the diffusivity. Figure 4 shows the same data as are shown by Figure 2 but plotted with the molecular diffusion model. Again the fit of the humidification data is better than that with the Schmidt number model.

Eddy Diffusion Model

Another model is obtained by assuming that the gas jet is analogous to the turbulent core for single-phase flow in a pipe and that the level of turbulence is such that molecular diffusion is negligible in comparison to eddy diffusion. It has been shown that eddy diffusion transfer in the core region can be represented by the Equation (10):

$$k_G = \frac{cu^*}{RT} \tag{10}$$

in which c is a slight function of the Reynolds number. If the pressure change for the gas phase is assumed equal to the liquid head

$$\frac{\Delta P}{\Delta l} = \frac{g}{g_c}\rho_L = \frac{2u^{*2}\rho_G}{rg_c} \tag{11}$$

substitution for u^* results in the equation

$$k_G = \frac{c}{RT}\sqrt{\frac{g\rho_L r}{2\rho_G}} \tag{12}$$

Consideration with respect to interfacial area, combination with Equation (6), and substitution for G give

$$N_G = \frac{c\bar{a}}{u_G}\sqrt{\frac{g\rho_L r}{2\rho_G}}$$

and substitution of $F = u_G\sqrt{\rho_G}$

$$N_G = c\bar{a}\sqrt{\frac{gr}{2}}\sqrt{\frac{\rho_L}{F}} \tag{13}$$

The group $c\bar{a}\sqrt{gr}$ can be expected to be a function of F. Figure 5 shows this correlation with the same data as are shown by Figures 2 and 4. Average absolute deviation of the experimental data from the least squares fit of the data is 13.1% for the molecular diffusion model and 11.6%

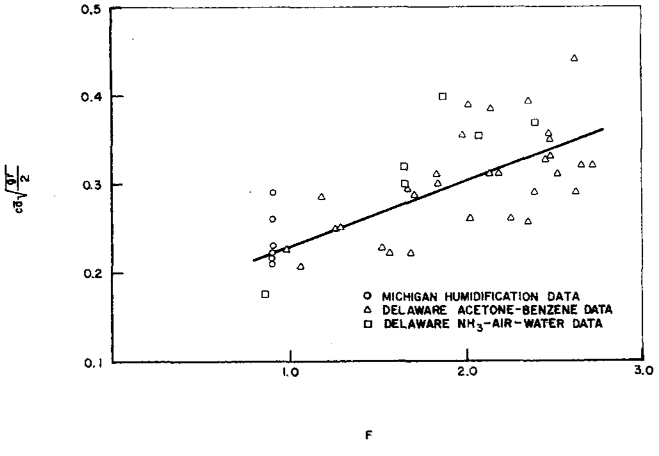

Fig. 5. Eddy diffusion model.

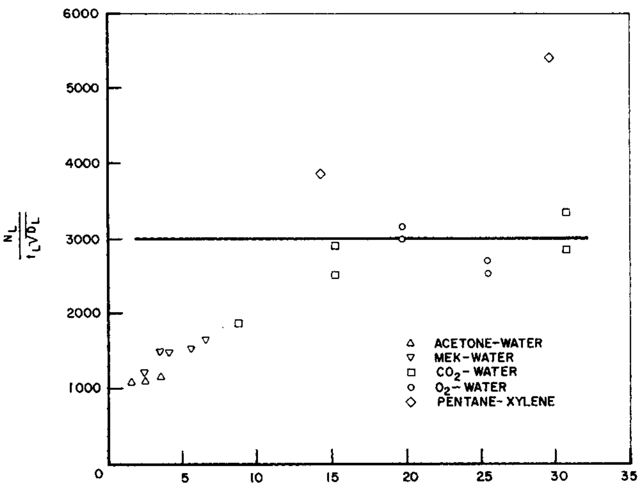

Fig. 6. AIChE liquid-phase model; $F = 1.2$ to 1.5.

for the eddy diffusion model. Both of these correlations are within the probable experimental error of the data.

LIQUID PHASE

The AIChE correlation for the number of liquid-phase transfer units is given by Equation (2). This equation indicates that the group $N_L/t_L\sqrt{D_L}$ should be constant for a constant value of F. Figure 6 shows experimental data for F of approximately 1.3 for which Equation (2) specifies that $N_L/t_L\sqrt{D_L} = 3{,}000$. It is apparent that the data for a low liquid flow rate represent values of N_L that are much less than the values predicted by Equation (2). This is consistent with the observation that liquid-phase-controlled binary systems operating at low liquid rates have much lower efficiencies than are predicted by the design method. Data shown by Figure 6 are for the carbon dioxide-water (*6*) and oxygen-water (*9*) systems in which essentially all of the resistance is in the liquid phase, and MEK-water (*11*), acetone-water (*12*), and pentane-xylene (*9*) which have significant vapor-phase resistance. The vapor-phase contribution for these data was estimated with the molecular diffusion model shown by Figure 4.

A molecular diffusion model would be expected to apply to the liquid phase at the gas jet interface because the turbulence level in the liquid phase is much less than in the gas stream. The Beek and Bakker analysis shows that the penetration theory applies for this condition, so the mass transfer coefficient is

$$k_L = 2\sqrt{\frac{D_L}{\pi t_L'}} \tag{14}$$

and for the interfacial area per unit of tray bubbling area

$$k_L\bar{a} = 2\bar{a}\sqrt{\frac{D_L}{\pi t_L'}} \tag{15}$$

and

$$N_L = \frac{k_L a \rho_L'}{L_M} \tag{16}$$

so combination Equations (15) and (16) gives

$$N_L = \frac{2\bar{a}\rho_L'}{L_M\sqrt{\pi t_L'}}\sqrt{D_L} \tag{17}$$

The group $2\bar{a}/\sqrt{\pi t_L'}$ can be considered as a function of F and the liquid rate. Figure 7 shows $N_L L_M/\rho_L'\sqrt{D_L}$ as a function of the liquid rate for an F of approximately 1.5. These include the data shown by Figure 6, and it is observed that the group $2\bar{a}/\sqrt{\pi t_L'}$ increases with liquid rate on the tray.

Experimental data for liquid-phase mass transfer require sample analysis for the inlet and outlet downcomers, analysis for the vapor leaving the tray, and a model for liquid mixing on the tray. The acetone-water, MEK-water, and carbon dioxide-water data were obtained on a rectangular tray with mixing data for each system. Tray sample data were obtained for the pentane-xylene runs so that mixing can be estimated for these data. The University of Michigan work also include experimental data for absorption of carbon dioxide in cyclohexanol that represent a liquid viscosity of about 60 centipoise. Tray analyses were taken with these data. Figure 7 shows the results from several runs with a liquid rate of 7.7 g.p.m./ft. tray width. These data are observed to fit the correlation for the lower liquid viscosity systems. Experimental data were also obtained for the F range of 0.9 to 1.8 with this system and are shown by Figure 8. The group $2\bar{a}/\sqrt{\pi t_L'}$ obtained from these data is observed to increase with F factor. Thus the group increases with liquid rate on the tray and with F factor. Both variables would be expected to increase turbulence in the liquid phase and the apparent result of this is indeed observed.

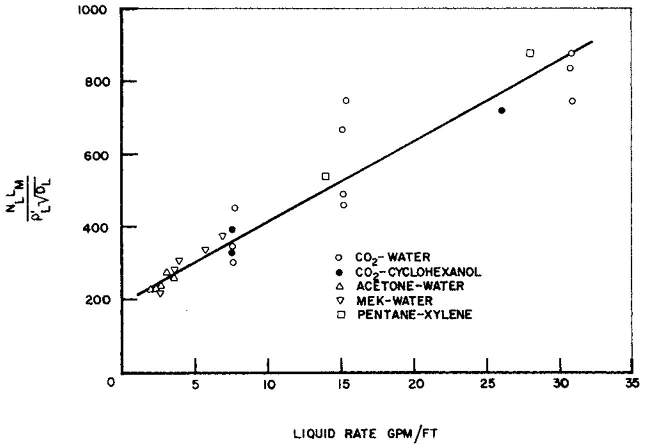

Fig. 7. Liquid-phase mass transfer; $F = 1.2$ to 1.5.

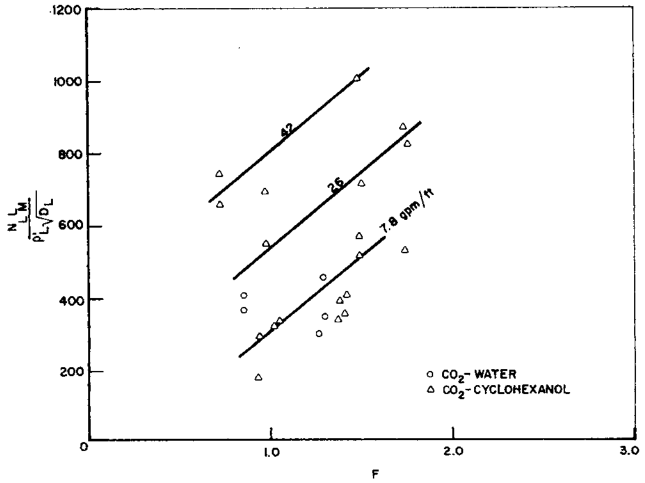

Fig. 8. Liquid-phase mass transfer.

DESIGN EQUATIONS

The molecular and eddy diffusion models appear to correlate the gas- or vapor-phase data from the AIChE program within the error of the data. Design equations can be obtained from these correlations with the eddy diffusion model providing a correlation that requires only the F factor and liquid density. Thus Equation (13) can be used as a design equation with dimensional empiricism to represent the group $c\bar{a}\sqrt{gr/2}$ as shown by Figure 5.

$$c\bar{a}\sqrt{\frac{gr}{2}} = 0.184 + 0.0463\,F \qquad (18)$$

Thus the gas- or vapor-phase number of mass transfer units can be estimated from the equation

$$N_G = (0.184 + 0.0463\,F)\sqrt{\frac{\rho_L}{F}} \qquad (19)$$

Equation (19) should be restricted to $\sqrt{\rho_L/F} < 9$ in accordance with the jet model and with the experimental data used for this correlation.

A liquid-phase correlation is indicated by Equation (17), and Figures 7 and 8. Thus dimensional empiricism for F and the liquid rate represents the group $2\bar{a}/\sqrt{\rho t_L'}$ from Equation (17).

$$\frac{2\bar{a}}{\sqrt{\pi t_L'}} = -44 + 22.3\,L + 155\,F \qquad (20)$$

Substitution in Equation (17) provides an estimate for N_L:

$$N_L = (-44 + 22.3\,L + 155\,F)\,\frac{\rho_L'\sqrt{D_L}}{L_M} \qquad (21)$$

The range of experimental data represented by Equation (21) is $L < 32$ and $0.9 < F < 2$.

NOTATION

$\bar{a}$ = interfacial area per unit of bubbling area, sq.ft./sq.ft.
c = constant in Equation (10), dimensionless
D_G = gas-phase diffusivity, sq.ft./sec.
D_L = liquid-phase diffusivity, sq.ft./sec.
F = F factor, $u_G\sqrt{\rho_G}$, lb.$_m^{1/2}$/(sec.)(ft.)$^{1/2}$
G = gas flow rate per unit of bubbling area, lb.-mole/(sec.)(sq.ft.)
g = acceleration due to gravity, ft./sec.^{2}
g_c = conversion factor, (ft.)(lb.$_m$)/(lb.$_f$)(sec.^{2})
k_G = gas-phase mass transfer coefficient, lb.-mole/(sec.)(sq.ft.)(atm.)
k_L = liquid-phase mass transfer coefficient, ft./sec.
L = liquid flow rate, gal./(min.)(ft.)
L_M = liquid flow rate per unit of bubbling area, lb.-mole/(sec.)(sq.ft.)
l = length of gas jet, ft.
N_G = number of gas-phase tranfer units
N_L = number of liquid-phase transfer units
N_{Sc} = Schmidt number
P = pressure, atm.
R = gas constant
r = radius of jet, ft.
T = temperature, °R.
t_G' = gas- or vapor-phase eddy contact time, sec.
t_L = liquid residence time on tray, sec.
t_L' = liquid-phase eddy contact time, sec.
u_c^+ = u_c/u^*
u_c = average velocity of masses of fluid exchanging momentum with the liquid interface, ft./sec.
u_G = gas velocity based on bubbling area, ft./sec.
u^* = $\sqrt{\tau_0\, g_c/\rho}$, ft./sec.
W = weir height, in.

Greek Letters

μ = fluid viscosity, lb.$_m$/(ft.)(sec.)
ρ_G = gas density, lb.$_m$/cu.ft.
ρ_L = liquid density, lb.$_m$/cu.ft.
ρ_L' = liquid density, lb.-mole/cu.ft.
τ_0 = shear stress at interface, lb.$_f$/sq.ft.

SUMMARY

Analysis of mass transfer data for bubble-cap distillation trays indicates that a jet model is applicable to the data for the F range of 0.9 to 2.7. This model assumes that gas or vapor leaves the bubble-cap slots as a jet and that the liquid at the jet interface is in concurrent flow with the jet. The analysis shows that a model based upon a rigid interface for the gas phase yields the Schmidt number as a correlating variable and that the experimental data do not fit this model. This is the AIChE design model. Free interface models are considered for molecular and eddy diffusions for the gas side of the jet interface. The molecular diffusion model shows that the group $2\bar{a}/\sqrt{\pi t_G'}$ is a linear function of F and increases with F. The eddy diffusion model assumes that molecular diffusion is negligible in comparison to eddy diffusion at the gas side of the jet interface. This model is found to give a slightly better fit with the experimental data than does the molecular diffusion model. The liquid-phase analysis shows that the AIChE design equation based upon oxygen-water data does not extrapolate to low liquid rate data. A free interface model for the liquid side of the interface shows that the group $2\bar{a}/\sqrt{\pi t_L'}$ is a function of F and the liquid rate on the tray.

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